| ieformation potentials at $L$. |  |
| :---: | :---: |
| zation | Zero of bands above: |
| . 32 eV | $E_{d}=5.75 \mathrm{sil}$ |
|  | e |

a the relative change of the vel:
of $\epsilon_{2}$ at 4.05 eV , whict. ilarity of $W_{11}+2 W_{12}$. changes of $M$ and $J$ whi 2 only do not contrib. core, $W_{i j}$ has its maximu $t$ and where the contri: total $\epsilon_{2}$ is still small. would have the large contribution of $L_{2}{ }^{\prime} \rightarrow$ es of $M$ and $J$ is justif analysis of the previo the effect of shear stri:
, $L_{1}$ selection rules a re $\mathbf{k}=\mathbf{L}$ is parallel With spin, these rules w $\left(\left|M_{z^{\prime}}\right|^{2} \ll\left|M_{x^{\prime}}\right|^{2}\right)$. T je different from the or n. The strong localizatic ssures that this deviatic of the transition will iations from the selecti

Is determined from e ing the assumption d: $\partial c=(-9.6 \pm 1.5) \mathrm{eV}$ ar eV for $\mathbf{k}$ parallel [111] lese coefficients is due the values given earlie: ckground slope used wa used here).

## ion Potentials at $L$

the deformation poter: s given earlier ${ }^{12}$ neglecte the wave function of $t$ t tion. The treatment ou' idization.
uiltonian developed $b$ workers, ${ }^{33}$ and Mueller uce, edited by W. A. Harris pons, Inc., New York, 1960
Phys. Letters 10, 203 (1965, D. Lang, Phys. Rev. 15:

659 (1967).

Table VI. Deformation potentials (in eV) of the transitions at $L$.

| Normation <br> stentials | Experiment |  | Theory |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | Present paper | Zallen ${ }^{\text {a }}$ | Present paper | $\mathrm{DFJ}^{\mathrm{b}}$ | Jacobs ${ }^{\text {e }}$ |
| $\begin{array}{r} \left.-E_{p}\right) / \partial e_{y_{z}} \\ \left.-E_{P}\right) / \partial^{d} \\ \left.-L_{a}^{u}\right) / \partial e \end{array}$ | $\begin{aligned} & -72 \pm 12 \\ & =9.6 \pm 1.5 \\ & -1.1 \pm 0.1 \end{aligned}$ | $9.7 \pm 2.0$ Absolute value $<1.3$ | $3^{-56}$ | $\begin{aligned} & -5.1 \\ & -0.8 \end{aligned}$ |  |
| $\left.\therefore \therefore-L_{3}{ }^{*}\right) / \partial e$ |  |  |  | -5.9 | -4.1 |

- Reference 47.

Reterence 43 .
$\rightarrow$, Recrence 48 .
the form given by Cohen and Mueller. ${ }^{35}$ Using symctrized plane waves as well as symmetrized tightding functions, we can write down the eigenvalues of $\vdots$ and of $L_{3}{ }^{u, l}$ immediately:

$$
\begin{align*}
L_{2}{ }^{\prime} & =k^{2}-V_{111}  \tag{7}\\
L_{3}{ }^{l, u} & =E_{d}+\pi-\delta \pm\left[(\pi-\delta)^{2}+4.5(-\sigma+\delta)^{2}\right]^{1 / 2} \tag{8}
\end{align*}
$$

the vector k is that of the $L$ point, $V_{111}$ is a pseudovential form factor, $E_{d}$ gives the position of the $d$ unds above $\Gamma_{1}$, and $\sigma \equiv(d d \sigma), \pi \equiv(d d \pi)$, and $\delta \equiv(d d \delta)$ tee the two-center tight-binding integrals defined by Iter and Koster. ${ }^{36}$ The two $L_{1}$ levels are obtained om the secular equation

$$
\left|\begin{array}{ll}
H_{\Phi \Phi}-E & H_{\Phi d}  \tag{9}\\
H_{d \Phi} & H_{d d}-E
\end{array}\right|=0 .
$$

The function $d$ is a tight-binding Bloch sum symetrized to $L_{1}$ and $\Phi$ is a plane wave symmetrized to t and orthogonalized to $d$,

$$
\begin{equation*}
\Phi=\left(\varphi-b_{d} d\right) / C . \tag{10}
\end{equation*}
$$

The abbreviations used in Eq. (10) are

$$
\begin{align*}
\varphi & =(2 / V)^{1 / 2} \cos (\mathbf{k} \cdot \mathbf{r}),  \tag{11}\\
b_{d} & =\langle\varphi \mid d\rangle,  \tag{12}\\
C^{2} & =1-b_{d}{ }^{2} . \tag{13}
\end{align*}
$$

The matrix elements of Eq. (9) are

$$
\begin{align*}
& H_{d d}=E_{d}-4(\pi-\delta),  \tag{14}\\
& H_{\Phi d}=\left(H_{\varphi d}-b_{d} H_{d d}\right) / C  \tag{15}\\
& H_{\Phi \Phi}=\left(k^{2}+V_{111}+V_{\varphi \varphi}(2)+b_{d}^{2} H_{d d}-2 b_{d} H_{\varphi d}\right) / C^{2} . \tag{16}
\end{align*}
$$

We calculated $b_{d}$ and the tight-binding integrals using the atomic wave function and the atomic potential alculated by Hartree and Hartree ${ }^{37}$ and parametrized

[^0]

Fig. 13. The dependence of the shear strain deformation potential $\partial L_{1} / \partial e_{y z}$ on the strain coefficient of the orthogonality integral $b_{d}$ [part (a) of the figure] and on the strain coefficient of the hybridization $H_{\varphi d}$ [part (b) of the figure]. The value $\partial b_{d} /$ $\partial e_{\nu z}=0.73$ was calculated using atomic $d$ functions.
by Fletcher and Wohlfahrth. ${ }^{38}$ The numerical values are given in Table V. The tight binding integrals agree with those calculated by Fletcher and Wohlfahrth. The value of the orthogonalization integral $b_{d}$ given by Mueller ${ }^{34}$ is $16 \%$ lower than the one reported here.
A first-principles calculation of the quantities $E_{d}$, $H_{\varphi d}$, and $\Delta V_{\varphi \varphi}{ }^{(2)}$ is extremely difficult and will not be attempted here. Instead, we determine them from the eigenvalues of Fig. 9, ${ }^{13,14}$ using the calculated values of $b_{d}$ and of the tight-binding integrals. In particular, the value of the hybridization integral $H_{\varphi d}$ is evaluated from the difference between $H_{d d}$ (the eigenvalue of $L_{1}{ }^{d}$, neglecting hybridization) and $L_{1}{ }^{d} . E_{d}$ is calculated from $L_{3}-\Gamma_{1}$ using Eq. (8). ( $L_{3}{ }^{u}-L_{3}{ }^{l}$ given by this equation agrees with the value taken from Fig. 9. One would expect this, since these bands have no interaction with the $s p$ bands.) The form factor $V_{111}$ given by Eq. (7) is also taken from the calculated band structure.
$\Delta V_{\varphi \varphi}{ }^{(2)}$ is the matrix element of the crystal potential, calculated with the $l=2$ component of $\varphi$. It was introduced in the model Hamiltonian ${ }^{35}$ following a suggestion by Heine. ${ }^{39}$ Its numerical value (calculated using the $L_{1}-L_{1}{ }^{d}$ gap of Fig. 9) is small, namely, -0.75 eV . Neglecting $\Delta V_{\varphi \varphi}{ }^{(2)}$ gives $L_{1}-L_{1}{ }^{d}=9.85 \mathrm{eV}$ using Eq. (9), which is only 0.60 eV higher than the value of Fig. 9. $\Delta V_{\varphi \varphi}{ }^{(2)}$ will be neglected in the following. The zero-strain values of $V_{111}, H_{\varphi d}$, and $E_{d}$ are listed in Table V.
The tight-binding parameters for changed nearestneighbor distances were calculated in the same way as for the distance in the unstrained crystal. For trigonal distortion (Table I) and levels with k parallel to [111], the strain coefficient of $H_{d d}$ is ( $R$ is the nearest-neighbor

[^1]
[^0]:    ${ }^{2}$ M. H. Cohen and F. M. Mueller, in Atomic and Electronic Thucture of Metals (American Society for Metals, Metals Park, Thio, 1967), p. 61.
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[^1]:    ${ }^{38}$ G. C. Fletcher and E. P. Wohlfahrth, Phil. Mag. 42, 106 (1951).
    ${ }^{39}$ V. Heine, Phys. Rev. 153, 673 (1967).

